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Integral root of single variable Polynomials:

Polynomial equations with one variable, x, can be checked if the value of x is an integer.

Let D_1 = { p | p is a polynomial over x with an integral root }

Example 1: $3x^2 + 2x - 56 = 0$ is in the set D_1. This polynomial belongs to D_1, because x = 4.

Example 2: $6x^2 + 13x + 6 = 0$ does **NOT** belong to D_1 (x = -1.5 (or x= -2/3 = -.666))

(13.5 - 19.5 +6 = 0 with x= -1.5) [Factor example2 as (3x +2)(2x +3) = 0]

The set D_1 is decidable. "Here is a TM, M_1, that recognizes D_1:

M_1 = "The input is a polynomial over the variable x.

1. Evaluate p with x set successively to the values 0, 1, -1, 2, -2, 3, -3, ...
   If at any point the polynomial evaluates to 0, *accept*"

For one variable polynomials, M_1 terminates (becomes decider), because the roots of such a polynomial must be between the values:

$$\pm (k * C_{max} / C_1)$$

Where k is the number of terms in the Polynomial, C_{max} is the coefficient of the largest absolute value, and C_1 is the coefficient of the highest order term.

For example 1, $3x^2 + 2x - 56 = 0$

The roots of this Polynomial must be between: $\pm (3 * 56/3) = +-56$

This polynomial belongs to D_1, because x = 4.

For multivariable Polynomials,

Let D = { p | p is a polynomial with integral roots } is ***undecidable****; that is, on some inputs the program may run for ever.*

This was the 10th problem on David Hilbert's list.

In 1900, in recognition of their depth, David Hilbert proposed the solvability of all Diophantine problems as the tenth of his celebrated problems. In 1970, a novel result in mathematical logic known as Matiyasevich's theorem settled the problem negatively: in general, Diophantine problems are unsolvable.

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